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Implications of Some Static Spherically Symmetric Graviton-Dilaton Solutions in Brans-Dicke and Low Energy String Theory[1]

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ABSTRACT. Analysing the static, spherically symmetric graviton-dilaton solutions in low energy string and Brans-Dicke theory, we find the following. For a charge neutral point star, these theories cannot predict non trivial PPN parameters, β and γ, without introducing naked singularities. We then couple a cosmological constant Λ as in low energy string theory. We find that only in low energy string theory, a non zero Λ leads to a curvature singularity, which is much worse than a naked singularity. Requiring its absence upto a distance r_* implies a bound $|\Lambda| < 10^{-102}(\frac{r_*}{\mathrm{pc}})^{-2}$ in natural units. If $r_* \simeq 1$Mpc then $|\Lambda| < 10^{-114}$ and, if $r_* \simeq 10^{28}$cm then $|\Lambda| < 10^{-122}$ in natural units.

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1. We study the static spherically symmetric solutions for Brans-Dicke (BD) and low energy string theory, including only the graviton and the dilaton field. They describe the gravitational field of a charge neutral point star, in these theories. Calculating the parametrised post Newtonian (PPN) parameters β and γ [1], we find that all the acceptable solutions predict $\beta = \gamma = 1$, the same as in Einstein's theory. There are more general static spherically symmetric solutions [2]-[6] predicting $\beta = 1,\ \gamma = 1 + \epsilon$, but they always have naked curvature singularities proportional to ϵ^2 and, hence, are unacceptable.

These general solutions can be better understood by coupling the electromagnetic field [3, 4]. They lead to non trivial PPN parameters for a point star of charge Q . In these solutions there is an inner and an outer horizon. The curvature scalar is singular at the inner horizon, but this singularity is hidden behind the outer horizon. A charge neutral star can then be obtained in two ways: in one, corresponding to the Schwarzschild solution, the PPN parameters are trivial and there is no naked singularity, while in the other, the PPN parameters are non trivial but there is a naked singularity.

Therefore neither BD nor low energy string theory can predict non trivial values for PPN parameters β and γ, for a charge neutral star, without introducing naked singularities.

We also couple a cosmological constant Λ, in a way analogous to the coupling of a tree level cosmological constant in low energy string theory [7]. The static spherically symmetric solutions here describe the gravitational field of point stars, and it is reasonable to expect them to be valid upto a distance r_*, of $\mathcal{O}(\mathrm{pc})$, even when the real universe is not static but expanding.

From an analysis of these solutions, we find [10] that for low energy string theory, a non zero Λ leads to a curvature singularity which is physically unacceptable. Requiring their absence imposes a bound $|\Lambda| < 10^{-102}(\frac{r_*}{\mathrm{pc}})^{-2}$ in natural units. Thus if $r_* \simeq 1\mathrm{Mpc}$ then $|\Lambda| < 10^{-114}$, and if r_* extends all the way upto the edge of the universe (10^{28}cm) then $|\Lambda| < 10^{-122}$ in natural units. For more details see [8]. For static solutions in other contexts, see [9]

2. Consider the following action for graviton ($\tilde{g}_{\mu\nu}$) and dilaton (ϕ) fields,

$$S = -\frac{1}{16\pi} \int d^4x \sqrt{\tilde{g}}\, e^{\phi}\, (\tilde{R} - \tilde{a}(\tilde{\nabla}\phi)^2 + \Lambda) \tag{1}$$

where Λ is the cosmological constant coupled to ϕ, as in string theory, and

$R_{\mu\nu\lambda\tau} = \frac{\partial^2 g_{\mu\lambda}}{\partial x^\nu \partial x^\tau} + \cdots$. For low energy string theory $\tilde{a} = 1$, and for BD theory $\tilde{a} = -\omega$, the BD parameter.

It is easier to solve the equations of motion if one makes the transformation $\tilde{g}_{\mu\nu} = e^{-\phi} g_{\mu\nu}$. The physical curvature scalars is given by

$$\tilde{R} = e^{\phi}(R - 3\nabla^2\phi + \frac{3}{2}(\nabla\phi)^2) \tag{2}$$

where R is the curvature scalar obtained using $g_{\mu\nu}$. The equations of motion become

$$\begin{aligned} 2R_{\mu\nu} + a\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\Lambda e^{-\phi} &= 0 \\ a\nabla^2\phi + \Lambda e^{-\phi} &= 0\,, \end{aligned} \tag{3}$$

where $a \equiv 3 - 2\tilde{a}$. We study the static, spherically symmetric solutions to equations (3).

If $\Lambda = 0$, we take the metric to be $ds^2 = -f dt^2 + f^{-1} d\rho^2 + r^2 d\Omega^2$, where the fields f, r, and ϕ depend only on ρ. If $\Lambda \neq 0$, we take the metric to be $ds^2 = -f dt^2 + \frac{G}{f} dr^2 + r^2 d\Omega^2$, where the fields f, G, and ϕ depend only on r. In these expressions, $d\Omega^2$ is the line element on an unit sphere. In the first case, the equations (3) become

$$\begin{aligned} \frac{(fr^2)''}{2} - 1 &= (f'r^2)' \\ = a(\phi' f r^2)' - \Lambda_\phi r^2 e^{-\phi} &= -\Lambda r^2 e^{-\phi} \\ 4r'' + ar\phi'^2 &= 0 \end{aligned} \tag{4}$$

where $'$ denotes ρ-derivatives. In the second case, they become

$$\begin{aligned} \frac{(fr^2)''}{2} - \frac{(fr^2)'G'}{4G} - G &= (f'r^2)' - \frac{G'f'r^2}{2G} \\ = a(\phi' f r^2)' - \frac{a\phi' G' f r^2}{2G} - \Lambda_\phi G r^2 e^{-\phi} &= -\Lambda G r^2 e^{-\phi} \\ 2G' - arG\phi'^2 &= 0 \end{aligned} \tag{5}$$

where $'$ denotes r-derivatives, and the physical curvature scalar $\tilde{R}$ is given by

$$\tilde{R} = \frac{(3-a) f \phi'^2 e^{\phi}}{2G} + \frac{(3-2a)\Lambda}{a}\,. \tag{6}$$

By writing the physical metric $\tilde{g}_{\mu\nu}$ in isotropic form, and expanding its tt and rr components asymptotically, one obtains the PPN parameters. For details, see [1].

3. When $\Lambda = 0$, the most general solutions to equations (4) are given by [2, 6]

$$f = \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{1-k^2}{1+k^2}} \ , \ r^2 = \rho^2 \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{2k^2}{1+k^2}} \ , \ e^{\phi - \phi_0} = \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{2l}{1+k^2}} \tag{7}$$

where k is a parameter and $l \equiv \frac{k}{\sqrt{a}}$. Writing the physical metric $\tilde{g}_{\mu\nu}$ in the isotropic gauge, $d\tilde{s}^2 \equiv -\tilde{f} dt^2 + \tilde{F}(d\rho^2 + \rho^2 d\Omega^2)$, where r and ρ are related by

$$\rho = h \left(1 + \frac{\rho_0}{4h}\right)^2 \ , \tag{8}$$

the physical mass M and the PPN parameters β and γ are given, after a straightforward calculation, by

$$2M = \frac{1 - k^2 - 2l}{1 + k^2} \rho_0 \ , \ \ \beta = 1 \ , \ \ \gamma = 1 + \frac{2l\rho_0}{(1 + k^2)M} \ . \tag{9}$$

The parameter β is trivial while γ is non trivial if $l\rho_0 \neq 0$. The physical curvature scalar $\tilde{R}$ is given by

$$\tilde{R} = \frac{\tilde{a} M^2 (\gamma - 1)^2 e^{\phi_0}}{\rho^4} \ \left(1 - \frac{\rho_0}{\rho}\right)^{-\frac{1+3k^2-2l}{1+k^2}} \ . \tag{10}$$

In the above equations ρ_0 is positive, so that one obtains the standard Schwarzschild solution when $k = 0$. Also the physical mass M, given by (9), must be positive which then implies that $1 - k^2 - 2l > 0$. Hence, the metric component $\tilde{g}_{tt}$ in the physical frame vanishes at $\rho = \rho_0$. The above condition on k also implies that $1 + 3k^2 - 2l > 0$. Hence, the curvature scalar $\tilde{R}$ in (10) becomes singular there, unless $\gamma = 1$, *i.e.* unless the PPN parameters are trivial. This singularity is naked, as will be shown presently.

The experimentally observed range of the PPN parameter γ is $\gamma = 1 \pm .002$. Requiring $|\gamma - 1| < \epsilon < .002$, and taking into account the constraint

$1-k^2-2l>0$, restricts k to be

$$|k| < \frac{\epsilon\sqrt{a}}{2(1+\epsilon)} \ . \tag{11}$$

Now we will discuss the nature of the singularity at $\rho=\rho_0$.

1. As can be seen from equation (10), the curvature scalar is singular at $\rho=\rho_0$; hence, this singularity is not a coordinate artifact and cannot be removed by any coordinate transformation.
2. The metric on the surface $\rho=\rho_0$ has the signature $0+++$, and hence, this surface is null and the singularity is a null one.
3. Consider an outgoing radial null geodesic, which describes an outgoing photon. Since $d\tilde{s}^2=0$ for such a geodesic, its equation is given by

$$\frac{dt}{d\rho} = \left(1-\frac{\rho_0}{\rho}\right)^{\frac{k^2-1}{k^2+1}} \ ,$$

where t is the external time, which gives $t=\rho_*+const$, where ρ_*, the analog of the 'tortoise coordinate', is defined by

$$\rho_* = \int d\rho \left(1-\frac{\rho_0}{\rho}\right)^{\frac{k^2-1}{k^2+1}} \ .$$

For $k\neq 0$, it is easy to show that $\rho_*(\rho)$ is finite. The outgoing radial null geodesic equation given above then implies that a radially outgoing photon starting from ρ_i $(\geq \rho_0)$ at external time t_i will reach an outside observer at ρ_f $(\rho_i<\rho_f<\infty)$ at a finite external time t_f given by $t_f-t_i=\rho_*(\rho_f)-\rho_*(\rho_i)$. Hence, it follows that a photon can travel from arbitrarily close to the singularity to an outside observer within a finite external time interval and, therefore, the singularity at $\rho=\rho_0$ is naked.

For these reasons, the singularity at $\rho=\rho_0$ is naked and physically unacceptable. For recent detailed discussions on naked singularities and their various general aspects see [11]).

One can gain more insight into the solution (7) by coupling a $U(1)$ gauge field A_μ, as in [3, 4], with field strength $F_{\mu\nu}$. The general solution is then given by

$$f = \left(1-\frac{\rho_1}{\rho}\right)\left(1-\frac{\rho_0}{\rho}\right)^{\frac{1-k^2}{1+k^2}} \ , \ \ r^2=\rho^2\left(1-\frac{\rho_0}{\rho}\right)^{\frac{2k^2}{1+k^2}}$$

$$e^{\phi-\phi_0} = \left(1-\frac{\rho_0}{\rho}\right)^{\frac{2l}{1+k^2}} \quad , \quad F_{t\rho} = \frac{Q}{\rho^2} \tag{12}$$

where $l = \frac{k}{\sqrt{a}}$ and the remaining components of $F_{\mu\nu}$ are zero. Writing the physical metric $\tilde{g}_{\mu\nu}$ in the isotropic gauge as before, the physical parameters M, Q, β, and γ are given by

$$\begin{aligned} 2M &= \rho_1 + \frac{1-k^2-2l}{1+k^2}\rho_0 \; , \quad Q^2 = \frac{\rho_1\rho_0}{1+k^2} \\ \beta &= 1 + \frac{(1-l)Q^2}{2M^2} \; , \quad \gamma = 1 + \frac{2l\rho_0}{(1+k^2)M} \; . \end{aligned}$$

The parameter β is non trivial if the charge $Q \neq 0$ while γ is non trivial if $l\rho_0 \neq 0$.

The curvature scalar $\tilde{R}$ in the physical frame is given by

$$\tilde{R} = \frac{\tilde{a}M^2(\gamma-1)^2 e^{\phi_0}}{\rho^4} \left(1-\frac{\rho_1}{\rho}\right) \left(1-\frac{\rho_0}{\rho}\right)^{-\frac{1+3k^2-2l}{1+k^2}} \; . \tag{13}$$

The metric component $\tilde{g}_{tt}$ in the physical frame vanishes at $\rho = \rho_1$ and $\rho = \rho_0$. The curvature scalar $\tilde{R}$ is regular at $\rho = \rho_1$ but, since $1 + 3k^2 - 2l > 0$ for $a \geq 1$, it is singular at $\rho = \rho_0$ unless $\gamma = 1$. This singularity is hidden behind the horizon at ρ_1 if $\rho_1 > \rho_0$, and naked otherwise for the same reasons as given following equation (10).

Now, a charge neutral solution *i.e.* $Q = 0$, can be obtained by setting either $\rho_0 = 0$ or $\rho_1 = 0$. The first case corresponds to the Schwarzschild solution while the second one, to the solution (7) for which γ is non trivial.

Thus, it follows that in BD or low energy string theory, a non trivial value for γ for a charge neutral point star implies the existence of a naked singularity. Conversely, in these theories, the absence of naked singularities necessarily implies that the PPN parameters β and γ for a charge neutral point star are trivial.

4. In the presence of both the dilaton ϕ, and a non zero cosmological constant $\Lambda \neq 0$, the solution to equations (5) is not known in an explicit form. Here we study this solution and its implications. Any solution, required to reduce to the Schwarzschild one when $\Lambda = 0$, has the following general features:

(i) The dilaton field ϕ cannot be a constant. The only exception is when $\Lambda = \lambda e^{\phi}$, which corresponds to Einstein theory with a cosmological constant λ and a free field ϕ.

(ii) In equations (5), $\ln G$ and, hence G, strictly increases since $a \geq 1$ and consequently $(\ln G)' > 0$.

(iii) Consider the following polynomial ansatz for the fields as $r \to \infty$.

$$f = Ar^k + \cdots \ , \quad G = Br^l + \cdots \ , \quad e^{-\phi} = e^{-\phi_0} r^m + \cdots \tag{14}$$

where $\cdots$ denote subleading terms in the limit $r \to \infty$. Substituting these expressions into equations (5) gives, to the leading order, $2l = am^2$ and

$$\begin{aligned} \frac{(k+2)}{2}(k+1-\frac{l}{2})Ar^k - Br^l &= k(k+1-\frac{l}{2})Ar^k \\ = -am(k+1-\frac{l}{2})Ar^k &= -B\Lambda e^{-\phi_0} B r^{l+m+2} \ . \end{aligned} \tag{15}$$

The solution turns out to be $(k, l, m) = (2a, 2a, -2)$ or $(2, \frac{2}{a}, -\frac{2}{a})$, and

$$k(k+1-\frac{l}{2})A = -\Lambda e^{-\phi_0} B \ , \quad \left((m + \frac{2}{a})\Lambda e^{-\phi_0} + \frac{4}{r^{m+2}} \right) B = 0 \ .$$

It is easy to see that if $a > 1$, as in BD theory, then there is always a non trivial asymptotic solution with non zero A and B. Also, the physical curvature scalar $\tilde{R}$ for this solution is finite as $r \to \infty$. Therefore it is plausible that a full solution can be constructed with this asymptotic behaviour, which reduces to the Schwarzschild solution when $\Lambda = 0$.

However, if $a = 1$ as in low energy string theory, then the above equations are consistent only if $A = B = 0$. Hence, in this case, equations (5) do not admit a non trivial solution where the fields are polynomials in r as $r \to \infty$. A similar analysis will rule out asymptotic polynomial-logarithmic solutions, *i.e.* where the fields behave as $r^m(\ln^n r)(\ln^p \ln r) + \cdots$ as $r \to \infty$. Thus, for low energy string theory, the solutions cannot have such asymptotic behaviour.

From now on, let $a = 1$. For small r, the solutions to (5) are given by

$$\begin{aligned} f &= 1 - \frac{r_0}{r} - \frac{\Lambda r^2}{6} - \frac{\Lambda^2 r^4}{120} u_2 + \cdots \\ G &= 1 + \frac{\Lambda^2 r^4}{72} v_2 + \cdots \\ \phi &= \phi_0 - \frac{\Lambda r^2}{6}(1 + \frac{2r_0}{r} + \frac{2r_0^2}{r^2}\ln(r - r_0)) - \frac{\Lambda^2 r^4}{45} w_2 + \cdots \end{aligned} \tag{16}$$

where ϕ_0 is a constant which can be set to zero, and $u_i,\ v_i,\ w_i$ are functions of $\frac{r_0}{r}$ and $\ln r$ which tend to 1 in the limit $\frac{r_0}{r} \ll 1$. Evaluating further higher order terms will not illuminate the general features of the solution. Also, the series will typically have a finite radius of convergence beyond which it is meaningless. Hence we follow a different approach.

It turns out that one can understand the general features of the solutions using only the equations (5), the behaviour of the fields for small r, and their non polynomial-logarithmic behaviour as $r \to \infty$.

Note that $G = 1$ for Schwarzschild solution. Let G has no pole at any finite r. Then the requirement that any solution to (5) reduce to the Schwarzschild one when $\Lambda = 0$, combined with the fact that G is a non decreasing function, implies that $G(\infty)$ and, hence, B must be non zero. Then the above analysis, which excludes polynomial behaviour for the fields with non trivial coefficients, implies in particular, that the fields cannot be constant, including zero, as $r \to \infty$.

Consider first the case where $r_0 = 0$. From equations (5), one then gets gives $e^\phi = |f|$. Also (see (16)), f has a local maximum (minimum) at the origin if Λ is positive (negative). Away from the origin, the function f can (A) have no pole at any finite r and go to either ∞ or a constant as $r \to \infty$, or (B) have a pole at a finite $r = r_p$ (its behaviour for $r > r_p$ will not be necessary for our purposes). We will also consider the case where (C) f has a zero at $r = r_H$.

Case A: The function f, and hence G, has no pole at finite r. From the above analysis, it follows that $f(\infty)$ cannot be a constant. Hence, $f(\infty) \to \infty$, which also follows from the asymptotic non polynomial-logarithmic behaviour of f.

Whether these singularities are genuine or only coordinate artifacts can be decided by evaluating the curvature scalar, $\tilde{R}$, or equivalently $R_1 \equiv \frac{f\phi'^2 e^\phi}{G}$ which can be shown, using (5), to obey the equation

$$R_1' + \frac{4R_1}{r} = -2\Lambda \frac{f'}{f} \ . \tag{17}$$

Now, $R_1(\infty)$ cannot be a constant. For, if it were, then one gets $f(\infty) \to r^{-\frac{2R_1(\infty)}{\Lambda}}$, a polynomial behaviour for f as $r \to \infty$, which is ruled out. Equa-

tion (17) can be solved to give

$$R_1 = -\frac{2\Lambda}{r^4} \int dr \frac{r^4 f'}{f} \; .$$

From this it follows, as $r \to \infty$, that $\frac{f'}{f} > \frac{k}{r}$ for any constant k (otherwise $R_1(\infty) \to \mathit{constant}$). This implies that $R_1(\infty) \to \infty$.

Case B: The function f has a pole at a finite $r = r_p < \infty$. Then, from equation (17) it follows, near $r = r_p$, that

$$R_1(r_p) = -2\Lambda \ln f(r_p) + \mathcal{O}(r - r_p) \quad \to \quad \pm\infty \; .$$

Case C: The function f has a zero at $r = r_H$. Then, from equation (17) it follows, near $r = r_H$, that

$$R_1(r_H) = -2\Lambda \ln f(r_H) + \mathcal{O}(r - r_H) \quad \to \quad \pm\infty \; .$$

Thus we see that R_1, and hence, the curvature scalar $\tilde{R}$ in the string frame, always diverges at one or more points $r \equiv r_s = r_p, \; r_H, \; \infty$, in low energy string theory when the cosmological constant $\Lambda \neq 0$. These singularities, which will persist even when $r_0 \neq 0$ as argued below, are naked. In fact, they are much worse, as they are created by any object, no matter how small its mass is. Thus at any point of the string target space, there will be a singularity produced by an object located at a distance r_s from that point.

When $r_0 \neq 0$ one can repeat the above analysis. Now, one starts at an $r > r_0$, and where $\frac{r_0}{r} < \frac{\Lambda r^2}{6}$. Then, the analysis proceeds as before. If Λ is positive (negative), then the function f will be decreasing (increasing), as r increases beyond $(\frac{6 r_0}{|\Lambda|})^{\frac{1}{3}}$. One then considers cases (A), (B), and (C) as before, arriving at the same conclusion. This is also physically reasonable since the cosmological constant can be thought of as vacuum energy density and, as r increases, the vacuum energy overwhelms any non zero mass of a star, which is proportional to r_0.

Thus, when the cosmological constant $\Lambda \neq 0$, the static spherically symmetric gravitational field of a point star in low energy string theory has a curvature singularity, much worse than a naked singularity.

Requiring their absence upto a distance r_* then imposes a constraint on Λ. If we take, somewhat arbitrarily, that the curvature becomes unacceptably strong when $|\Lambda| r^2 \simeq 1$, then $|\Lambda| r_*^2 < 1$, and we get the bound

$$|\Lambda| < 10^{-102} (\frac{r_*}{\mathrm{pc}})^{-2}$$

in natural units. Thus if $r_* \simeq 1$Mpc then $|\Lambda| < 10^{-114}$, and if r_* extends all the way upto the edge of the universe (10^{28}cm) then $|\Lambda| < 10^{-122}$ in natural units.

5. We have analysed the static, spherically symmetric solutions to the graviton-dilaton system, with or without electromagnetic couplings and the cosmological constant. These solutions describe the gravitational field of a point star. The main results are as follows.

1. For a charge neutral point star, neither BD nor low energy string theory predicts non trivial PPN parameters, β and γ, without introducing naked singularities.

2. With a cosmological constant Λ, coupled as in in low energy string theory, the static spherically symmetric solutions are likely to exist for BD type theories, with no naked singularities. However, for low energy string theory, a non zero Λ leads to a curvature singularity, much worse than a naked singularity. Requiring the absence of this singularity upto a distance r_* implies a bound $|\Lambda| < 10^{-102}(\frac{r_*}{\mathrm{pc}})^{-2}$ in natural units. If $r_* \simeq 1$Mpc then $|\Lambda| < 10^{-114}$ and, if $r_* \simeq 10^{28}$cm then $|\Lambda| < 10^{-122}$ in natural units.

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